تم الرفع بواسطة
م. معن ابو عيسى

Calculus 1
Second

Palestine Technical University
Mathematics Department.
Calculus I
Dr. Khalida Nazzál

33/60

Student # [redacted]

Second Exam

Dec. 7[th], 2009

Student name بالعربية: معن سلام قعدان

Sec #: RTN 11:00 - 12:00

Part I: Choose the correct answer (30 points)

1. The displacement of a particle moving in a straight line is given by $s=\frac{t^4}{4}-8t+3$.

When the velocity of the particle is equal to zero.

(a) $t=4$
(b) $t=0$
(c) $t=1$
(d) $t=2$

$\frac{4t^3}{4}-8$ $t^3-8=0$, $t^3=8 \Rightarrow t=\sqrt[3]{8}=2$

2. If the sides of a cube changes from 2 to 2.01, the new volume is approximately

(a) 0.12.
(b) 8.12
(c) 8.00
(d) 7.88.

$dx=0.01$ $V=x^3=2^3=8$ $dV=3x^2dx=3(2)^2(0.01)$ 0.12 $8+0.12=8.12$

3. The equation of the tangent line to the curve $x^3-xy+y^3=7$ at $P(2,1)$ is.

(a) $5(y-1)=11(x-2)$.
(b) $y=11x-21$.
(c) $(y-1)=-11(x-2)$.
(d) else.

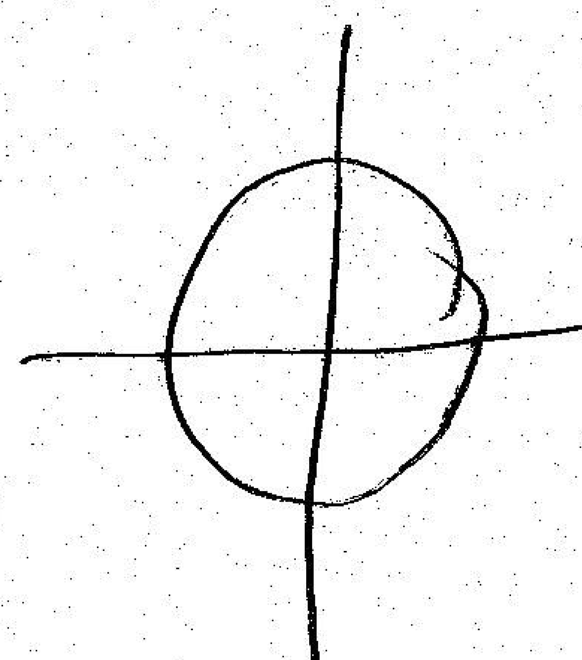
$3x^2+x\frac{dy}{dx}+y+3y^2\frac{dy}{dx}=0$ $3x^2+y=-x\frac{dy}{dx}-3y\frac{dy}{dx}$ $3x^2+y=-(x+3y)\frac{dy}{dx} \Rightarrow \frac{dy}{dx}=\frac{3x^2+1}{-x-3y}$ $\frac{dy}{dx}=\frac{3(2)^2+1}{-2-3}$ $\frac{16}{-5}$ $\frac{13}{-5}$

$y-1=\frac{18}{-5}(x-2)$ $y-1=\frac{-16}{5}x+\frac{32}{5}$ $5y-5=-16x+32$

4. Find the slope of the tangent line to the curve $x=2+\sin t$, $y=3+\cos t$, at $t=\frac{\pi}{4}$.

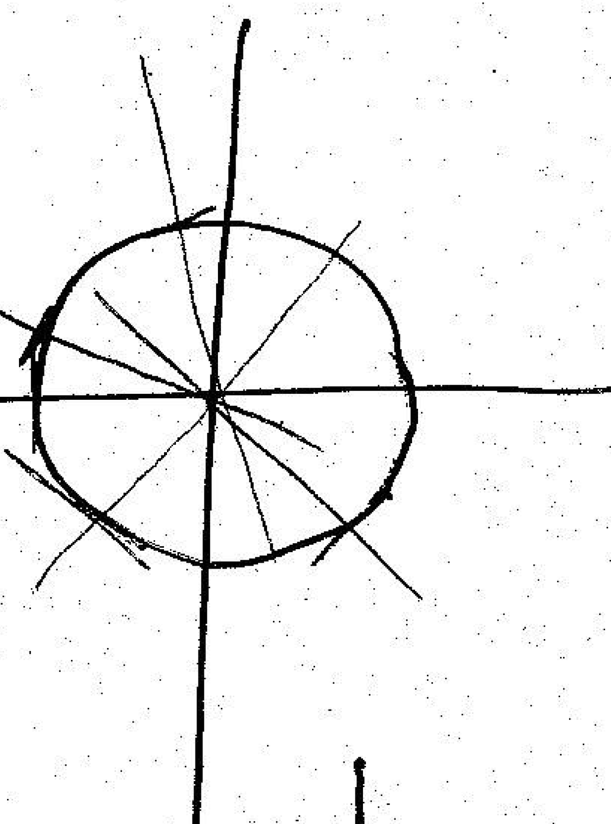
(a) 1
(b) 0
(c) -1
(d) not defined

$\sin t=x-2$ $\cos t=y-3$ $\sin^2t+\cos^2t=1$ $(x-2)^2+(y-3)^2=1$ $x^2-4x+4+y^2-6y+9=1$

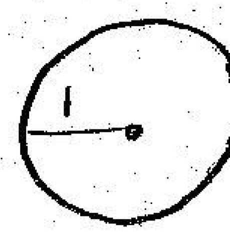


5. The normal line at any point of the circle $x^2+y^2=a^2$

(a) passes through the origin.
(b) has a negative slope
(c) is never a vertical line.
(d) is never a horizontal line.



$2(x-2)+3(y-3)y'=0$ $2x-4+3yy'-3y'=0$ $2x-4=3y'-3yy'$ $y'(3-3y)=2x-4$ $y'=\frac{2x-4}{3-3y}$



15

Page 1

~~x = -1~~ ($y = 2$)

6. Let x and y be functions of time t such that $x^2 + y^3 = 9$.

If $\frac{dx}{dt} = 5$, find $\frac{dy}{dt}$ when x=-1.

(a) $\frac{5}{8}$

(b) $\frac{5}{12}$

(c) $\frac{1}{2\sqrt{2}}$

(d) $\frac{5}{6}$ ✓

$2x\frac{dx}{dt} + 3y^2\frac{dy}{dt} = 0$

$2(-1)(5) + 3y^2\frac{dy}{dt} = 0$

$10 = 3y^2\frac{dy}{dt}$

$x^2 = 9 - y^3$

$1 = 9 - y^3$

$y^3 = 9 - 1 = 8$

$y = 2$

$2(-1)(5) + 3(4)\frac{dy}{dt} = 0$

$-10 + 12\frac{dy}{dt} = 0$

$10 = 12\frac{dy}{dt} = \frac{10}{12}$ $\frac{5}{6}$

7. Given that $f(x) = 5 - \frac{4}{x}$, $x \in [2,4]$. Find the value(s) of c in the conclusion of the Mean Value Theorem.

(a) $2\sqrt{2}, -2\sqrt{2}$

(b) 1

(c) $2\sqrt{2}$ ✓

(d) 2,-2

$f'(c) = \frac{f(b) - f(a)}{b - a}$

$5 - 4x^{-1}$

$= 4x^{-2} = \frac{4}{x^2}$ $\quad \frac{4}{c^2} = \frac{4-3}{4-2} \Rightarrow \frac{4}{c^2} = \frac{1}{2}$ $\quad c^2 = 8$

$c = \sqrt{8}$

$\pm 2\sqrt{2}$

8. If $h(x) = g(f(x))$ and $f(2)=5$, $g(5)=3$, $f'(2) = 7$, and $g'(5) = -2$, find $h'(2)$.

(a) -14

(b) 21

(c) -10

(d) 14.

$h'(2) = g'(f(2)) = g'(5)$

$h'(2) = g'(f(2))\,f'(2)$

$h'(2) = g'(5)(7) = -2(7) = -14$

9. A parametrization of the left half of the parabola $y = x^2 + 2x$ could be

(a) $x = t^2,\ y = t^4 + 2t^2,\ ,-\infty < t < \infty$.

(b) $x = t,\ y = t^2 + 2t,\ ,-\infty < t < \infty$.

(c) $x = \sin t,\ y = \sin^2 t + 2\sin t,\ ,-\infty < t < \infty$.

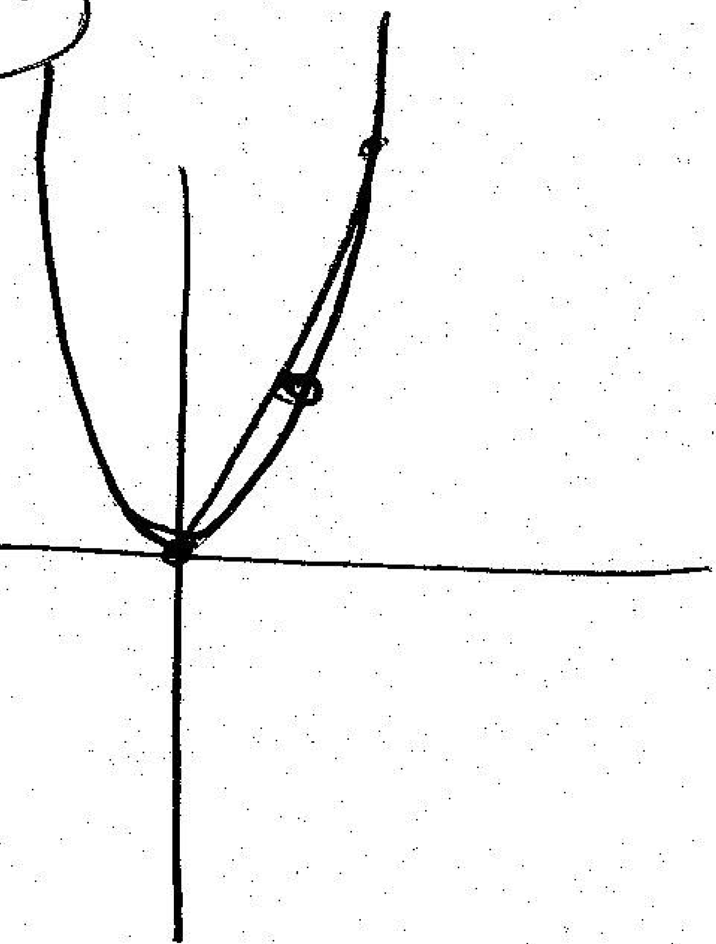
(d) $x = t,\ y = t^2 + 2t,\ ,t \le -1$

$x = t^2$

$y = (t^2)^2 + 2t^2$

$t^4 + 2t^2$

t^2



10. The linearization of $f(x) = \sqrt{x^2 + 8}$ at x = -1 is

(a) $y = \frac{8}{3} - \frac{1}{3}x$

(b) $y = 1 + \frac{1}{3}x$

(c) $y = 1 + \frac{1}{2}x$

(d) None of the above

$f(x_0) + f'(x_0)(x - x_0)$

$f(x) = (x^2 + 8)^{\frac{1}{2}}$ $\quad f'(x) = \frac{1}{2}(x^2+8)^{-\frac{1}{2}}(2x)$

$= \frac{x}{\sqrt{x^2+8}}$

$3 + \frac{-1}{3}(x + 1)$

~~$3 - \frac{1}{3}x + 1$~~

$3 - \frac{1}{3}(x + 1)$

$3 - \frac{1}{3}x - 1$

$y = 2 - \frac{1}{3}x$

$y = 2 - \frac{1}{3}x$

$x^{\frac{1}{3}}$ $x^{\frac{2}{3}}$

II. (20 points). Let $f(x) = (\frac{5}{2} - x)\, x^{\frac{2}{3}}$.

(a) find the intervals on which f is increasing or decreasing.

$f'(x) = (\frac{5}{2} - x)\, \frac{2}{3} x^{-\frac{1}{3}} + x^{\frac{2}{3}}(-1) = \frac{10}{6} x^{-\frac{1}{3}} - \frac{2}{3} x^{\frac{2}{3}} - x^{\frac{2}{3}}$

$= \frac{5}{3\sqrt[3]{x}} - \frac{5}{3}\sqrt[3]{x^2} = \frac{5}{3}\left(\frac{1}{\sqrt[3]{x}} - \frac{\sqrt[3]{x^2}}{1}\right) = \frac{5}{3}\left(\frac{1-x}{\sqrt[3]{x}}\right) = 0$

6

$\therefore \frac{1-x}{\sqrt[3]{x}} = 0 \Rightarrow \boxed{x=1}$, DNE $\boxed{x=0}$

increasing $0 < x < 1$

decreasing $x > 1$, $x < 0$ $(1,\infty) \cup (-\infty, 0)$

(b) find the x-coordinates of the local extrema of f.

$f(0) = 0 \Rightarrow (0, 0) \Rightarrow$ local minimum

4 $f(1) = \frac{3}{2}$, $(1, 1.5) \Rightarrow$ local maximum

$\frac{x^5}{x^4}$

(c) find the intervals on which f is CU or CD.

$f'(x) = \frac{5}{3}\left(\frac{1-x}{\sqrt[3]{x}}\right) \Rightarrow f''(x) = \frac{\sqrt[3]{x}(-1) - (1-x)(\frac{1}{3}x^{-\frac{1}{3}})}{x^{\frac{2}{3}}}$

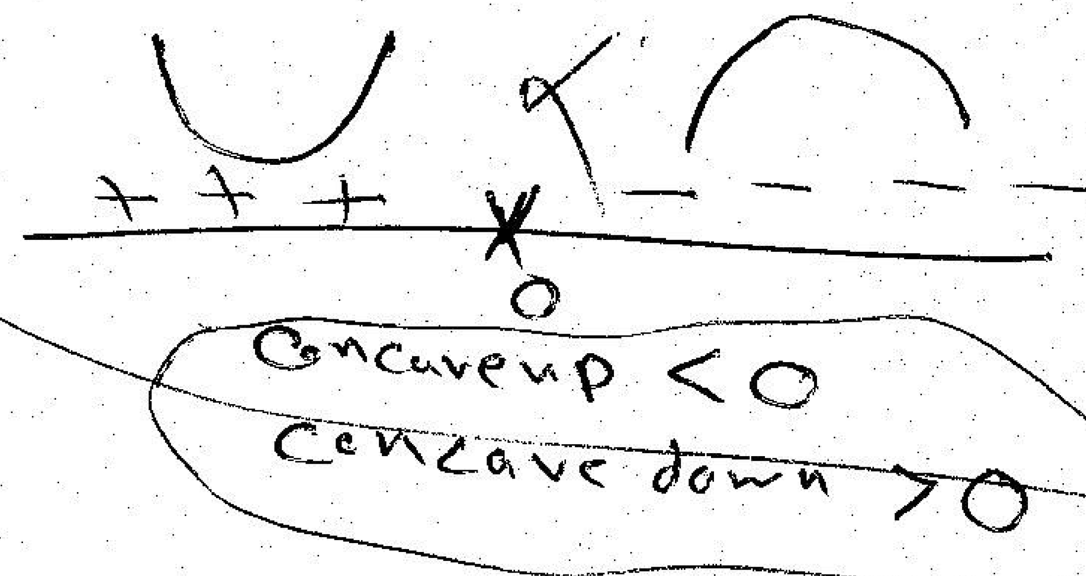
$= \frac{-\sqrt[3]{x} - [\frac{1}{3}x^{-\frac{1}{3}} - \frac{1}{3}x^{\frac{2}{3}}]}{x^{\frac{2}{3}}} = \frac{-\sqrt[3]{x} - \frac{1}{3}x^{-\frac{1}{3}} + \frac{1}{3}x^{\frac{2}{3}}}{x^{\frac{2}{3}}}$

$= \frac{-x^{\frac{1}{3}} - \frac{1}{3}x^{-\frac{1}{3}} + \frac{1}{3}x^{\frac{2}{3}}}{x^{\frac{2}{3}}} = -x^{-\frac{1}{3}} - \frac{1}{3}x^{-1} + \frac{1}{3} = 0$

$-\frac{1}{\sqrt[3]{x}} - \frac{1}{3x} + \frac{1}{3} = 0 \Rightarrow$ DNE at $x = 0$

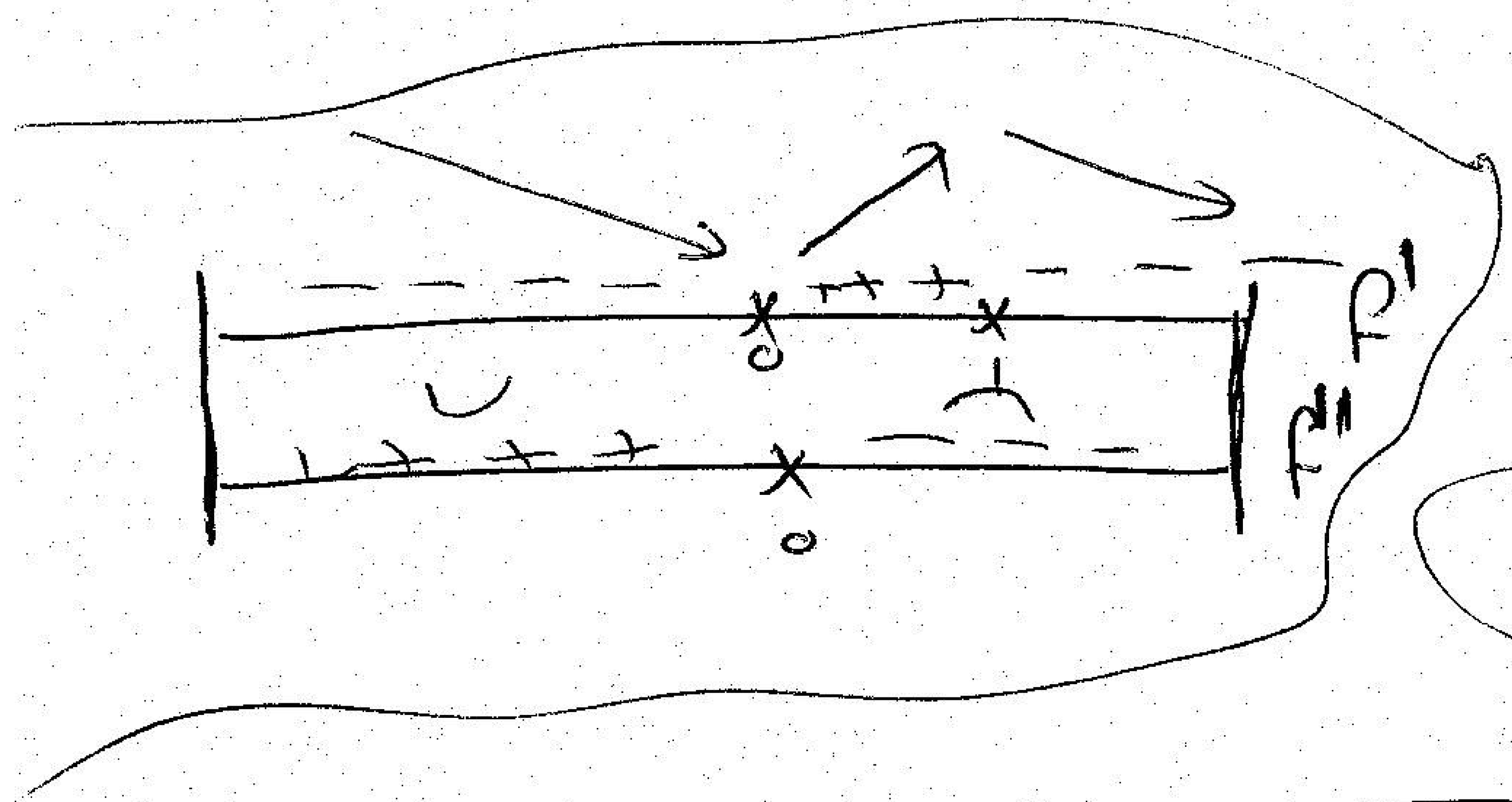
(d) find the x-coordinates of the points of inflection of f.

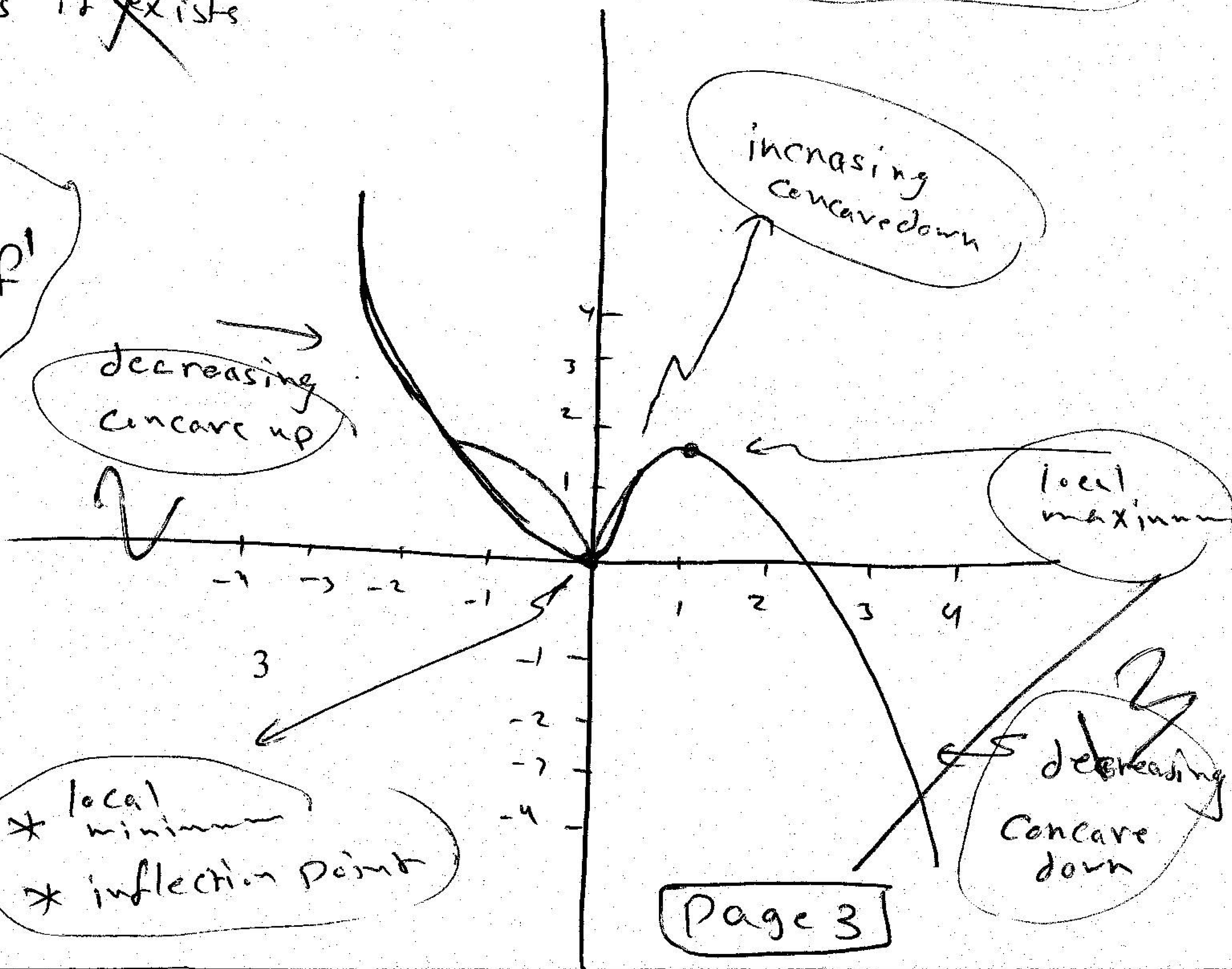
$f(0) = 0 \Rightarrow (0, 0)$



* find The cusp points if exists

(e) Sketch the graph of f.





III. A particle moves along the curve $y = x^{\frac{3}{2}}$ in the first quadrant in such a way that its distance from the origin increases at the rate 11 units per second. Find $\frac{dx}{dt}$ when $x = 3$.

(10 points).

$Y = X^{\frac{3}{2}}$, $\frac{ds}{dt} = 11$

distance $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\therefore$ distance $= \sqrt{x^2 + y^2} = \sqrt{x^2 + (x^{3/2})^2}$

(*) when $x = 3 \Rightarrow y = 3^{\frac{3}{2}} = \sqrt[2]{3^3} = \sqrt{27}$

~~$\frac{ds}{dt} = \frac{1}{2}(x^2+y^2)^{-\frac{1}{2}}(2x\,dx + 2y\,dy)$~~

NO!

$\therefore \quad S = \sqrt{x^2 + 27} = (x^2 + 27)^{\frac{1}{2}}$

$\therefore \frac{ds}{dt} = \frac{1}{2}(x^2 + 27)^{-\frac{1}{2}}(2x\,dx)$

$\therefore \quad 11 = \frac{1}{2}(3^2 + 27)^{-\frac{1}{2}}(2(3)\,dx)$

$11 = \frac{1}{2\sqrt{36}} \cdot (6\,dx)$

$11 = \frac{1}{12} \cdot 6(dx)$

$11 = \frac{1}{2}(dx) \Rightarrow dx = $ 22 units/second

5

Page 4